

10.1 Sequences and Summation Notation

Write the first four terms of the sequence whose general term is given.

1) **Exercise 7 pg 1010**

1) _____

$$a_n = (-1)^n (n + 3)$$

$$a_1 = (-1)^1 (1 + 3) = -4 \quad ; \quad a_2 = (-1)^2 (2 + 3) = 5$$

$$a_3 = (-1)^3 (3 + 3) = -6 \quad ; \quad a_4 = (-1)^4 (4 + 3) = 7$$

Write the first four terms of the sequence whose general term is given.

2) **Exercise 19 pg 1010**

2) _____

$$a_n = \frac{n^2}{n!}$$

$$a_1 = \frac{1^2}{1!} = \frac{1}{1} = 1 \quad ; \quad a_2 = \frac{2^2}{2!} = \frac{4}{2} = 2$$

$$a_3 = \frac{3^2}{3!} = \frac{9}{6} = \frac{3}{2} \quad ; \quad a_4 = \frac{4^2}{4!} = \frac{16}{24} = \frac{2}{3}$$

Evaluate the factorial expression.

3) **Exercise 27 pg 1010**

3) _____

$$\frac{(n + 2)!}{n!} = \frac{(n + 2)(n + 1)n!}{n!} = (n + 2)(n + 1)$$

Find the indicated sum.

4) **Exercise 39 pg 1010**

4) _____

$$\sum_{i=0}^4 \frac{(-1)^i}{i!} = \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!}$$

$$= \frac{1}{1} + \frac{-1}{1} + \frac{1}{2} + \frac{-1}{6} + \frac{1}{24} = \frac{9}{24} = \frac{3}{8}$$

Express the sum using summation notation.

5) **Exercise 49 pg 1010**

5) _____

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{14}{15}$$

The sum has fourteen terms, each of the form $\frac{i}{i+1}$, starting at $i = 1$ and ending at $i = 14$.

$$\text{Thus, } \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{14}{15} = \sum_{i=1}^{14} \frac{i}{i+1}.$$

Evaluate the factorial expression.

6) **Exercise 81 pg 1012**

6) _____

$$\frac{200!}{198!} = \frac{200 \cdot 199 \cdot 198!}{198!} = 200 \cdot 199 = 39,800$$

10.2 Arithmetic Sequence

Write the first four terms of the arithmetic sequence.

7) **Exercise 3 pg 1020**

$$a_1 = -7; d = 4$$

7) _____

$$a_2 = a_1 + d = -7 + 4 = -3.$$

$$a_3 = a_2 + d = -3 + 4 = 1.$$

$$a_4 = a_3 + d = 1 + 4 = 5.$$

Find the indicated term of the sequence with the given first term, a_1 , and common difference, d .

8) **Exercise 19 pg 1020**

8) _____

Find a_{200} when $a_1 = -40$, $d = 5$.

$$a_{200} = -40 + (200 - 1)(5) = -40 + (995) = 955$$

Write a formula for the general term (the n th term) of the arithmetic sequence. Then use the formula for a_n to find a_{20} , the 20th term of the sequence.

9) **Exercise 25 pg 1020**

9) _____

$$7, 3, -1, -5, \dots$$

$$a_n = a_1 + (n - 1)d = 7 + (n - 1)(-4) = 7 - 4n + 4 = -4n + 11.$$

$$a_{20} = -4(20) + 11 = -80 + 11 = -69.$$

Find the indicated sum.

10) **Exercise 36 pg 1020**

10) _____

Find the sum of the first 25 terms of the arithmetic sequence:

7, 19, 31, 43, ...

$$a_{25} = 7 + (25 - 1)(12) = 7 + 288 = 295.$$

$$S_{25} = \frac{25}{2}[7 + 295] = 3775.$$

Find the indicated sum.

11) **Exercise 41 pg 1021**

11) _____

Find the sum of the first 60 positive even integers.

$$a_{60} = 2 + (60 - 1)(2) = 2 + 118 = 120.$$

$$S_{60} = \frac{60}{2}[2 + 120] = 3660.$$

Find the indicated sum.

12) **Exercise 45 pg 1021**

12) _____

$$\sum_{i=1}^{17} (5i + 3) \quad a_1 = 5(1) + 3 = 8 \quad a_{17} = 5(17) + 3 = 88$$

$$S_{17} = \frac{17}{2}(8 + 88) = 816.$$

10.3 Geometric Sequence

Write the first four terms of the geometric sequence.

13) **Exercise 1 pg 1035**

13) _____

$$a_1 = 5; r = 3$$

$$a_2 = a_1 \cdot r = 5(3) = 15.$$

$$a_3 = a_2 \cdot r = 15(3) = 45. \quad a_4 = a_3 \cdot r = 45(3) = 135.$$

Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of the sequence with the given first term, a_1 , and common ratio, r .

14) **Exercise 11 pg 1035**

14) _____

Find a_{12} when $a_1 = 5$, $r = -2$.

$$a_{12} = (5)(-2)^{12-1} = (5)(-2)^{11} = -10,240.$$

Write a formula for the **general term (the nth term)** of the geometric sequence. Then use the formula for a_n to find a_7 , the 7th term of the sequence.

15) **Exercise 19 pg 1035**

15) _____

$$18, 6, 2, \frac{2}{3}, \dots$$

$$a_n = a_1 r^{n-1} = (18) \left(\frac{1}{3}\right)^{n-1}$$

$$a_7 = (18) \left(\frac{1}{3}\right)^6 = \frac{18}{729} = \frac{2}{81}$$

Find the indicated sum. Use the formula for the sum of the first n terms of a geometric sequence.

16) **Exercise 27 pg 1035**

16) _____

Find the sum of the first 11 terms of the geometric sequence:

$$3, -6, 12, -24, \dots$$

$$S_{11} = \frac{3(1 - (-2)^{11})}{1 - (-2)} = \frac{3(1 + 2048)}{1 + 2} = \frac{3(2049)}{3} = 2049.$$

Find the indicated sum. Use the formula for the sum of the first n terms of a geometric sequence.

17) **Exercise 33 pg 1035** $\sum_{i=1}^{10} 5(2)^i$

17) _____

$$S_{10} = \frac{10(1 - 2^{10})}{1 - 2} = \frac{10(1 - 1024)}{-1} = \frac{10(-1023)}{-1} = 10,230.$$

10.5 The Binomial Coefficient

Evaluate the given binomial coefficient.

18) **Exercise 1 pg 1053**

18) _____

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{40320}{6 \cdot 120} = 56$$

Use the Binomial Theorem to expand the binomial and express the result in simplified form.

19) **Exercise 15 pg 1053** $(2x + 1)^4$; $a = 2x$ $b = 1$

19) _____

$$\begin{aligned} &= \binom{4}{0} (2x)^4 (1)^0 + \binom{4}{1} (2x)^3 \cdot (1)^1 + \binom{4}{2} (2x)^2 \cdot (1)^2 \\ &+ \binom{4}{3} (2x)^1 \cdot (1)^3 + \binom{4}{4} (2x)^0 \cdot (1)^4 = 16x^4 + 32x^3 + 24x^2 + 8x + 1 \end{aligned}$$

Evaluate the given binomial coefficient.

20) **Exercise 33 pg 1054** $(x - 2y)^{10}$; $a = x$ $b = -2y$
$$= \binom{10}{0} (x)^{10} (-2y)^0 + \binom{10}{1} (x)^9 \cdot (-2y)^1 + \binom{10}{2} (x)^8 \cdot (-2y)^2$$
$$= x^{10} - 20x^9y + 180x^8y^2.$$

20) _____

Find the term indicated in the expansion.

21) **Exercise 39 pg 1054** $(2x + y)^6$; 3rd term
The 3rd term is: $\binom{6}{2} (2x)^4 \cdot (y)^2 = 15(16x^4)(y^2) = 240x^4y^2$

21) _____